17 - Bearing Capacity

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Bearing Capacity Factors for General Shear											
Angle φ (Degrees)	Angle φ (Radians)	Terzaghi				Meyerhof			Hansen		
		$K_{p\gamma}$	N_c	N_q	N_γ	N_c	N_q	N_γ	N_c	N_q	N_γ
0	0.0000	10.18	5.70	1.00	0.00	5.10	1.00	0.00	5.10	1.00	0.00
1	0.0175	10.61	6.00	1.10	0.08	5.38	1.09	0.00	5.38	1.09	0.00
2	0.0349	11.07	6.30	1.22	0.18	5.63	1.20	0.01	5.63	1.20	0.01
3	0.0524	11.56	6.62	1.35	0.28	5.90	1.31	0.02	5.90	1.31	0.02
4	0.0698	12.07	6.97	1.49	0.39	6.19	1.43	0.04	6.19	1.43	0.05
5	0.0873	12.61	7.34	1.64	0.51	6.49	1.57	0.07	6.49	1.57	0.07
6	0.1047	13.19	7.73	1.81	0.65	6.81	1.72	0.11	6.81	1.72	0.11
7	0.1222	13.80	8.15	2.00	0.80	7.16	1.88	0.15	7.16	1.88	0.16
8	0.1396	14.44	8.60	2.21	0.96	7.53	2.06	0.21	7.53	2.06	0.22
9	0.1571	15.13	9.09	2.44	1.15	7.92	2.25	0.28	7.92	2.25	0.30
10	0.1745	15.87	9.60	2.69	1.35	8.34	2.47	0.37	8.34	2.47	0.39
11	0.1920	16.65	10.16	2.98	1.58	8.80	2.71	0.47	8.80	2.71	0.50
12	0.2094	17.49	10.76	3.29	1.84	9.28	2.97	0.60	9.28	2.97	0.63
13	0.2269	18.38	11.41	3.63	2.12	9.81	3.26	0.74	9.81	3.26	0.78
14	0.2443	19.33	12.11	4.02	2.44	10.37	3.59	0.92	10.37	3.59	0.97
15	0.2618	20.36	12.86	4.45	2.79	10.98	3.94	1.13	10.98	3.94	1.18
16	0.2793	21.46	13.68	4.92	3.19	11.63	4.34	1.37	11.63	4.34	1.43
17	0.2967	22.65	14.56	5.45	3.63	12.34	4.77	1.66	12.34	4.77	1.73
18	0.3142	23.92	15.52	6.04	4.13	13.10	5.26	2.00	13.10	5.26	2.08
19	0.3316	25.30	16.56	6.70	4.70	13.93	5.80	2.40	13.93	5.80	2.48
20	0.3491	26.80	17.69	7.44	5.34	14.83	6.40	2.87	14.83	6.40	2.95
21	0.3665	28.42	18.92	8.26	6.07	15.81	7.07	3.42	15.81	7.07	3.50
22	0.3840	30.18	20.27	9.19	6.89	16.88	7.82	4.07	16.88	7.82	4.13
23	0.4014	32.10	21.75	10.23	7.83	18.05	8.66	4.82	18.05	8.66	4.88
24	0.4189	34.19	23.36	11.40	8.90	19.32	9.60	5.72	19.32	9.60	5.75
25	0.4363	36.49	25.13	12.72	10.12	20.72	10.66	6.77	20.72	10.66	6.76
26	0.4538	39.01	27.09	14.21	11.53	22.25	11.85	8.00	22.25	11.85	7.94
27	0.4712	41.78	29.24	15.90	13.15	23.94	13.20	9.46	23.94	13.20	9.32
28	0.4887	44.85	31.61	17.81	15.03	25.80	14.72	11.19	25.80	14.72	10.94
29	0.5061	48.26	34.24	19.98	17.21	27.86	16.44	13.24	27.86	16.44	12.84
30	0.5236	52.05	37.16	22.46	19.75	30.14	18.40	15.67	30.14	18.40	15.07
31	0.5411	56.29	40.41	25.28	22.71	32.67	20.63	18.56	32.67	20.63	17.69
32	0.5585	61.04	44.04	28.52	26.20	35.49	23.18	22.02	35.49	23.18	20.79
33	0.5760	66.40	48.09	32.23	30.33	38.64	26.09	26.17	38.64	26.09	24.44
34	0.5934	72.48	52.64	36.50	35.23	42.16	29.44	31.15	42.16	29.44	28.77
35	0.6109	79.40	57.75	41.44	41.08	46.12	33.30	37.15	46.12	33.30	33.92
36	0.6283	87.33	63.53	47.16	48.11	50.59	37.75	44.43	50.59	37.75	40.05
37	0.6458	96.49	70.07	53.80	56.62	55.63	42.92	53.27	55.63	42.92	47.38
38	0.6632	107.13	77.50	61.55	67.00	61.35	48.93	64.07	61.35	48.93	56.17
39	0.6807	119.59	85.97	70.61	79.77	67.87	55.96	77.33	67.87	55.96	66.76
40	0.6981	134.31	95.66	81.27	95.61	75.31	64.20	93.69	75.31	64.20	79.54
41	0.7156	151.89	106.81	93.85	115.47	83.86	73.90	113.99	83.86	73.90	95.05
42	0.7330	173.09	119.67	108.75	140.65	93.71	85.37	139.32	93.71	85.37	113.96
43	0.7505	198.99	134.58	126.50	173.00	105.11	99.01	171.14	105.11	99.01	137.10
44	0.7679	231.10	151.95	147.74	215.16	118.37	115.31	211.41	118.37	115.31	165.58
45	0.7854	271.57	172.29	173.29	271.07	133.87	134.87	262.74	133.87	134.87	200.81
46	0.8029	323.57	196.22	204.19	346.66	152.10	158.50	328.73	152.10	158.50	244.65
47	0.8203	391.94	224.55	241.80	451.28	173.64	187.21	414.33	173.64	187.21	299.52
48	0.8378	484.34	258.29	287.85	600.15	199.26	222.30	526.45	199.26	222.30	368.67
49	0.8552	613.53	298.72	344.64	819.31	229.92	265.50	674.92	229.92	265.50	456.40
50	0.8727	801.95	347.51	415.15	1155.97	266.88	319.06	873.86	266.88	319.06	568.57

The bearing capacity of a soil is its ability to carry loads without failing in shear. There are four major methods to predict failure. The fist method was developed by Karl Terzaghi in 1943. Field tests in Canada by Meyerhof (1963) lead to modification factors. Finally, Brinch Hansen in Denmark (1970) and Vesic in the USA modified these factor to a greater refinement.

These bearing capacity factors are based on these three authors:

Terzaghi (1943):

For square footings, $q_{ult} = 1.3c' N_c + \bar{q} N_q + 0.4 \gamma B N_\gamma$

For continuous or wall footings $q_{ult} = c' N_c + \bar{q} N_q + 0.5 \gamma B N_\gamma$

where, $\bar{q} = \gamma D_f$ *and the factors are,*

$$N_q = \frac{a^2}{a \cos^2(45° - \varphi/2)} \quad \text{where } a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

Meyerhof (1963):

For vertical loads, $q_{ult} = cN_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma}$

and for inclined loads, $q_{ult} = cN_c F_{ic} F_{dc} + \bar{q} N_q F_{iq} F_{dq} + 0.4 \gamma B N_\gamma F_{i\gamma} F_{d\gamma}$

and the factors are,

$$N_q = e^{\pi\tan\phi} \tan^2(45° - \phi/2)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Brinch Hansen (1970):

The general equation, $q_{ult} = cN_c F_{sc} F_{dc} F_{ic} + \bar{q} N_q F_{sq} F_{dq} F_{iq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma} F_{i\gamma}$

and the factors are,

$$N_q = e^{\pi\tan\phi} \tan^2(45° - \phi/2)$$

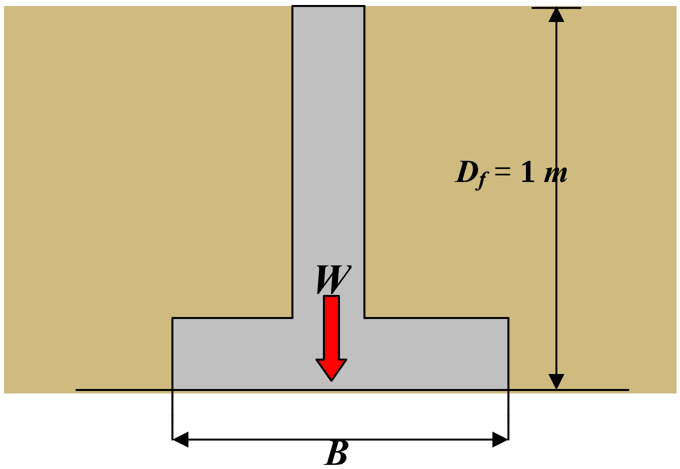
$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = 1.5(N_q - 1)\tan\phi$$

*Bearing–01: *Terzaghi's bearing capacity formula for a square footing.*

(Revision: Sept-08)

The square footing shown below must be designed to carry a 294 *kN* load. Use *Terzaghi's* bearing capacity formula to determine *B* of the square footing with a factor of safety =3.



Solution:

Terzaghi's formula for the ultimate bearing capacity q_{ult} of a square footing is,

$$q_{ult} = 1.3c' N_c + \bar{q} N_q + 0.4\gamma B N_\gamma \quad where \quad \bar{q} = D_f \gamma$$

The allowable bearing capacity q_{all} with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}\left(1.3c' N_c + \bar{q} N_q + 0.4\gamma B N_\gamma\right) \quad and \quad q_{all} = \frac{W}{B^2} = \frac{294\ kN}{B^2}$$

$$or \quad \frac{294}{B^2} = \frac{1}{3}\left(1.3c' N_c + \bar{q} N_q + 0.4\gamma B N_\gamma\right)$$

For ϕ=35º, N_c=57.8, N_q=41.4, and N_γ=41.1.

Substituting these values into Terzaghi's equation, we get

$$\frac{294}{B^2} = \frac{1}{3}\left[(0) + (18.15)(1)(41.4) + (0.4)(18.15)B(41.1)\right]$$

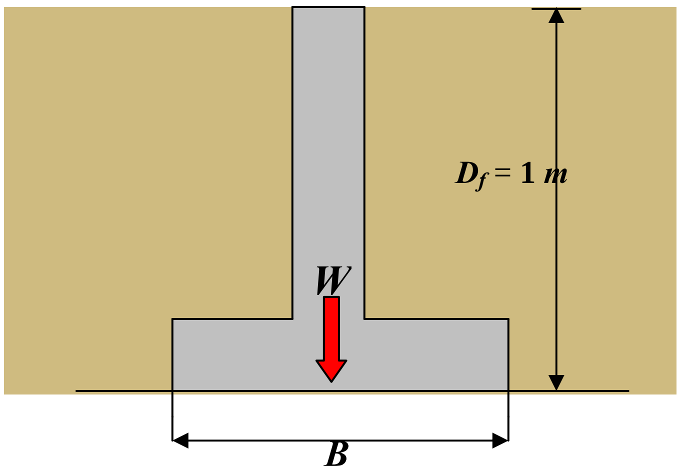
$$\frac{294}{B^2} = 250.5 + 99.5B$$

$$B^3 + 2.52B^2 - 2.96 = 0 \quad \therefore \quad B = 0.90\ m$$

*Bearing–02: Meyerhof's bearing capacity formula for a square footing.

(Revision: Sept-08)

The square footing shown below must be designed to carry a 294 *kN* load. Use Meyerhof's bearing capacity formula to determine *B* with a factor of safety =3.



W = 294 kN
γ = 18.15 kN/m^3
ϕ = 35°
c = 0

Solution:

Meyerhof's formula for the ultimate bearing capacity q_{ult} of a square footing is,

$$q_{ult} = c' N_c F_{sc} F_{dc} F_{ic} + \bar{q} N_q F_{sq} F_{dq} F_{iq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma} F_{i\gamma} \quad where \quad \bar{q} = D_f \gamma$$

Since the load is vertical, all three inclination factors $F_{ic} = F_{iq} = F_{i\gamma} = 1$.

$$F_{sq} = 1 + \left(\frac{B}{L}\right) \tan\phi = 1 + \left(\frac{1}{1}\right) \tan 35° = 1.70 \quad and \quad F_{s\gamma} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4(1) = 0.6$$

$$F_{dq} = 1 + 2\tan\phi\left(1 - \sin\phi\right)^2 \left(\frac{D_f}{B}\right) = 1 + 2\left(\tan 35°\right)\left(1 - \sin 35°\right)^2 \left(\frac{1}{B}\right) \approx 1.25 \quad and \ F_{d\gamma} = 1$$

The allowable bearing capacity q_{all} with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}\left(c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma}\right) \quad and \quad q_{all} = \frac{W}{B^2} = \frac{294\ kN}{B^2}$$

$$or \quad \frac{294}{B^2} = \frac{1}{3}\left(c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma}\right)$$

For ϕ = 35º, N_c = 46.12, N_q = 33.30, and N_γ = 37.15.

Substituting these values into Meyerhof"s equation, we get

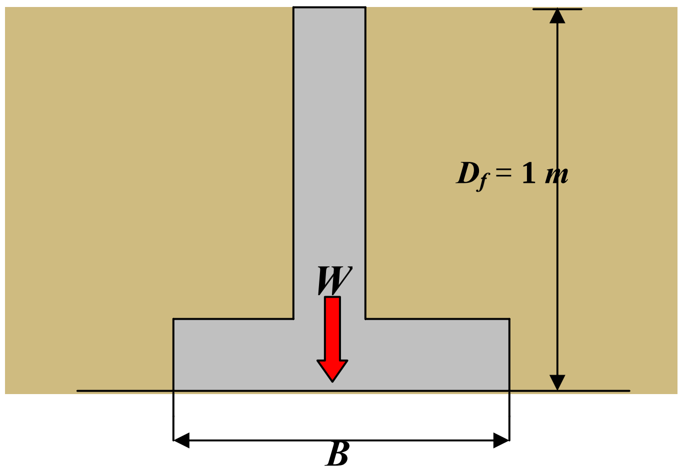
$$\frac{294}{B^2} = \frac{1}{3}\left[(0) + (18.15)(1)(33.3)(1.7)(1.25) + (0.4)(18.15)B(37.15)(0.6)(1)\right]$$

$$\frac{294}{B^2} = 428.1 + 53.94B \quad or \quad B^3 + 7.94B - 5.45 = 0 \quad \therefore B = \ 0.65\ m$$

*Bearing–03: Hansen's bearing capacity formula for a square footing.

(Revision: Sept-08)

The square footing shown below must be designed to carry a 294 *kN* load. Use Brinch Hansen's bearing capacity formula to determine *B* with a factor of safety =3.



$W = 294\ kN$
$\gamma = 18.15\ kN/m^3$
$\phi = 35°$
$c = 0$

Solution:

Hansen's formula for the ultimate bearing capacity q_{ult} of a square footing is,

$$q_{ult} = c' N_c F_{sc} F_{dc} F_{ic} + \bar{q} N_q F_{sq} F_{dq} F_{iq} + 0.4\gamma B N_\gamma F_{s\gamma} F_{d\gamma} F_{i\gamma} \quad where \quad \bar{q} = D_f \gamma$$

Since the load is vertical, all three inclination factors $F_{ic} = F_{iq} = F_{i\gamma} = 1$.

$$F_{sq} = 1 + \left(\frac{B}{L}\right)\tan\phi = 1 + \left(\frac{1}{1}\right)\tan 35° = 1.7 \quad and \quad F_{s\gamma} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4(1) = 0.6$$

$$F_{dq} = 1 + 2\tan\phi(1-\sin\phi)^2\left(\frac{D_f}{B}\right) = 1 + 2(\tan 35°)(1-\sin 35°)^2\left(\frac{1}{B}\right) \approx 1.255 \quad and \; F_{d\gamma} = 1$$

The allowable bearing capacity q_{all} with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}\left(c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4\gamma B N_\gamma F_{s\gamma} F_{d\gamma}\right) \quad and \quad q_{all} = \frac{W}{B^2} = \frac{294\ kN}{B^2}$$

$$or \quad \frac{294}{B^2} = \frac{1}{3}\left(c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4\gamma B N_\gamma F_{s\gamma} F_{d\gamma}\right)$$

For $\phi = 35°$, $N_c = 46.12$, $N_q = 33.30$, and $N_\gamma = 33.92$.

Substituting these values into Hansen's equation, we get

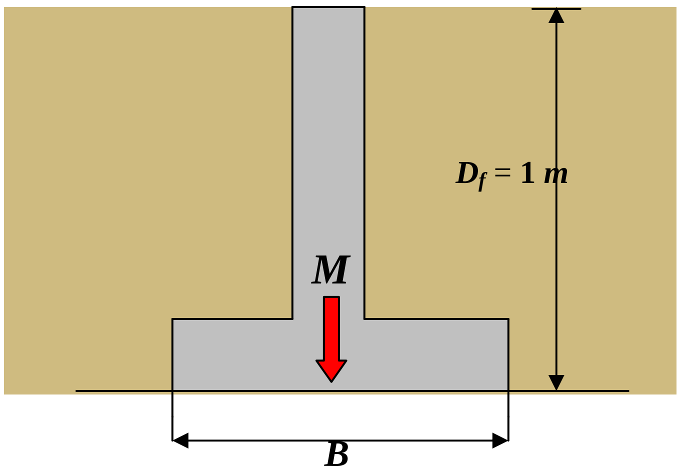
$$\frac{294}{B^2} = \frac{1}{3}\left[(0) + (18.15)(1)(33.3)(1.7)(1.255) + (0.4)(18.15)B(33.92)(0.6)(1)\right]$$

$$\frac{294}{B^2} = 429.8 + 49.25B \quad or \quad B^3 + 8.73B - 5.97 = 0 \quad \therefore B = 0.70\ m$$

*Bearing–04: *Same as #01 but requiring conversion from metric units.*

(Revision: Sept-08)

The square footing shown below must be designed to a load of 30,000 kg_m. Using a factor of safety of 3 and using *Terzaghi's* method, determine the size *B* of the square footing.



m = 30,000 kg_m
ρ = 1,850 kg/m^3
ϕ = 35°
c = 0

Solution:

The soil density $\rho = 1{,}850\ kgm/m^3$ converts to a unit weight via $\gamma\ =\ \rho g\ \ (like\ F = ma)$,

$$\gamma = \rho g = \frac{\left(1{,}850 \frac{kg_m}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)}{\left(1{,}000\ N/kN\right)} = 18.15\ kN/m^3$$ and the load to be supported by the footing is,

$$W = ma = \frac{\left(30{,}000\ kg_m\right)\left(9.81 \frac{m}{s^2}\right)}{\left(1{,}000\ N/kN\right)} = 294\ kN$$

Terzaghi's ultimate bearing capacity of a square footing is given by,

$$q_{ult}\ =\ 1.3c' N_c\ +\ \bar{q} N_q\ +\ 0.4\gamma B N_\gamma$$

$$\therefore q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}\left(1.3c' N_c + \bar{q} N_q + 0.4\gamma B N_\gamma\right) \quad and \quad q_{all} = \frac{P}{B^2} = \frac{294}{B^2}$$

$$or \quad \frac{294}{B^2} = \frac{1}{3}\left(1.3c' N_c + \bar{q} N_q + 0.4\gamma B N_\gamma\right)$$

For ϕ = 35°, N_c = 57.8, N_q = 41.4, and N_γ = 41.1,

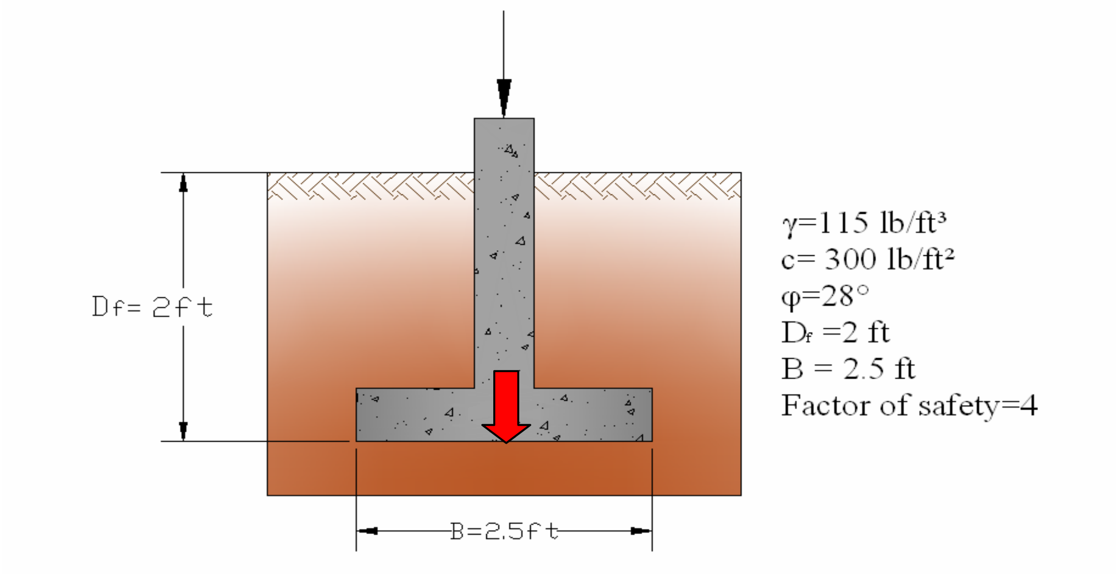
$$\frac{294}{B^2} = \frac{1}{3}\left[(0) + (18.15)(1)(41.4) + (0.4)(18.15)B(41.1)\right] \ \therefore B^3 + 2.52B^2 - 2.96 = 0$$

$B = 0.90\ m$

*Bearing–05: *General versus local bearing capacity failures.*

(Revision: Sept-08)

Using Terzaghi's method, distinguish between the value of the local shear failure versus the general shear failure.



Solution:

Terzahi's general bearing capacity failure of a square footing is,

$$q_{ult} = 1.3c' N_C + \bar{q}N_q + 0.4\gamma B N_\gamma$$

$$For\ \phi = 28°\ N_c = 31.6,\ N_q = 17.8,\ N_\gamma = 15.0\ and\ \bar{q} = \gamma D_f = (0.115)(2) = 0.23\ ksf$$

Therefore,

$$q_{ult} = 1.3(0.30)(31.6) + (0.23)(17.8) + 0.4(0.115)(2.5)(15.0) = 18.1\ ksf$$

To find the value of the bearing capacity of a local shear failure, the cohesion and angle of internal friction are reduced by two-thirds,

$$q'_{ult-local} = 1.3c' N'_c + \bar{q}N'_q + 0.4\gamma B N'_\gamma \quad where\ c' = \frac{2}{3}c = \frac{2}{3}(0.30) = 0.2\ ksf$$

$$and\ \varphi' = \frac{2}{3}(\varphi) = \frac{2}{3}(28°) = 18.7°\ which\ give\ N'_c = 16.2,\ N'_q = 6.5\ and\ N'_\gamma = 4.52$$

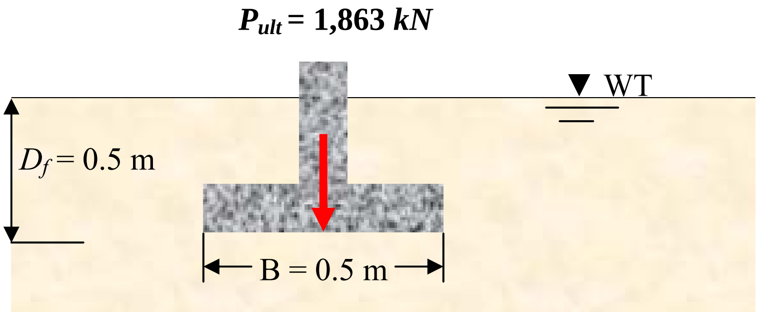
$$\therefore q'_{ult-local} = (1.3)(0.2)(16.2) + (0.23)(6.5) + (0.4)(0.115)(2.5)(4.52) = 6.2\ ksf$$

$$q_{ult-general\ failure} = 18.1\ ksf \quad versus \quad q_{ult-local\ failure} = 6.2\ ksf \quad (Almost\ a\ three-to-one)$$

*Bearing–06: Comparing the Hansen and Meyerhof bearing capacities.

(Revision: Sept-08)

Compare the results of the ***Hansen*** and the ***Meyerhof*** bearing capacity formulas to the results of a field test that took a rectangular footing to failure when the load reached 1,863 *kN*. Given $\boldsymbol{B}$ = 0.5 m, $\boldsymbol{L}$ = 2.0 ***m***, $\boldsymbol{c}$ = 0, $\varphi_{triaxial}$ = 42° and γ' = 9.31 $\boldsymbol{kN/m^3}$ (the *WT* is at the surface).



Solution:

$$q_{ult} = \frac{P_{ult}}{BL} = \frac{1,863\ kN}{(0.5\,m)(2.0\,m)} = 1,863\ kPa$$ was the field measured failure load.

(1) The Hansen formula predicts an ultimate bearing capacity of,

$$q_{ult} = 0 + \bar{q}N_q F_{qs} F_{qd} + 0.5\gamma B N_\gamma F_{\gamma s} F_{\gamma d}$$

$$Lee's\ adjustment\ formula\ is\ \phi_{ps} = 1.5\phi_{triaxial} - 17° = 1.5(42°) - 17° = 46°$$

$$For\ \varphi = 46°,\ N_q = 158.5\ and\ N_\gamma = 244.65$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\varphi = 1 + \left(\frac{0.5}{2}\right)\tan 46° = 1.26$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\left(\frac{0.5}{2}\right) = 0.9$$

$$F_{qd} = 1 + 2\tan\varphi(1-\sin\varphi)^2\left(\frac{D_f}{B}\right) = 1 + 2\tan 46°(1-\sin 46°)^2\left(\frac{0.5}{0.5}\right) = 1.16$$

$$F_{\gamma d} = 1.0$$

$$\therefore\quad q_{ult} = 0 + (9.31)(0.5)(159)(1.27)(1.16) + (0.5)(9.31)(0.5)(245)(0.9)(1.0)$$

$$\therefore\quad q_{ult} = 1,485\ kPa\ versus\ 1,863\ kPa\ measured\ (Hansen\ underestimates\ by\ 20\%)$$

(2) The Meyerhof formula with ϕ = 46º, N_q = 158.5 and N_γ = 328.73,

$$q_{ult} = 0 + \bar{q}N_q F_{qs} F_{qd} + 0.5\gamma B N_\gamma F_{\gamma s} F_{\gamma d}$$

$$\therefore\quad q_{ult} = 0 + (9.31)(0.5)(158.5)(1.27)(1.16) + (0.5)(9.31)(0.5)(328.73)(0.9)(1.0)$$

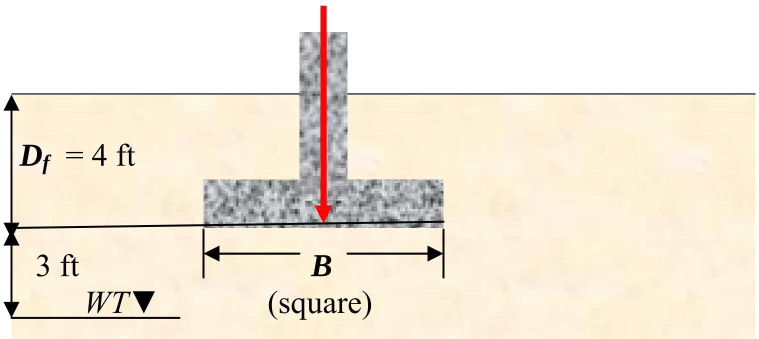
$$\therefore\quad q_{ult} = 1,782\ kPa\ versus\ 1,863\ kPa\ (Meyerhof\ underestimates\ by\ 4\ \%).$$

*Bearing–07: *Increase a footing's width if the WT is expected to rise.*

(Revision: Sept-08)

Use Meyerhof's bearing capacity formula (with a factor of safety = 3) to select a footing's width ***B*** if, (a) the water table is as shown below, and (b) if the water table rises to the ground surface? The soil has a unit weight of 112 pcf, a moisture of 10%, $\varphi = 25°$, a cohesion $c_u = 240$ *psf* and a specific gravity of solids of $G_s = 2.68$.

$$Q = 200 \;\; kips$$



Solution:

(a) Find γ_{SAT} to determine γ',

$$\gamma_{dry} = \frac{\gamma}{1 + w_N} = \frac{112}{1.10} = 101.8 \; pcf \;\; and \;\; V_s = \frac{W_s}{G_s \gamma_w} = \frac{\gamma_{dry}}{G_s \gamma_w}$$

$$set \; V = 1 \; ft^3 \; \therefore \; V_s = \frac{101.8}{2.68(62.4)} = 0.61 \; ft^3 \; \therefore V_v = V - V_s = 1 - 0.61 = 0.39 \; ft^3$$

$$but \; \gamma_{sat} = \gamma_{dry} + n\gamma_w = \gamma_{dry} + \left(\frac{V_v}{V}\right)\gamma_w \; \therefore \; \gamma_{sat} = 101.8 + (0.39)(62.4) = 126.2 \; pcf$$

$$and \; \gamma' = \gamma_{sat} - \gamma_w = 126.2 - 62.4 = 63.8 \; pcf$$

Try $B = 5.7$ feet with Meyerhof's equation,

$$q_{ult} = c' N_c (F_{cs} F_{cd} F_{ci}) + \overline{q} N_q (F_{qs} F_{qd} F_{qi}) + 0.5 \gamma B N_\gamma (F_{\gamma s} F_{\gamma d} F_{\gamma i})$$

where the load inclination factors F_{ci}, F_{qi} *and* $F_{\gamma i} = 1$

$$For \; \varphi \geq 10° \quad K_p = \tan^2\left(45° + \frac{\varphi}{2}\right) = \tan^2\left(45° + \frac{25°}{2}\right) = 2.46, \; therefore$$

$$F_{cs} = 1 + (0.2)\left(\frac{B}{L}\right)K_p = 1 + (0.2)\left(\frac{5.7}{5.7}\right)(2.46) = 1.49$$

$$F_{cd} = 1 + (0.2)\left(\frac{D_f}{B}\right)\sqrt{K_p} = 1 + (0.2)\left(\frac{4}{5.7}\right)\sqrt{2.46} = 1.22$$

$$F_{qd} = F_{\gamma d} = 1 + (0.1)\left(\frac{D_f}{B}\right)\sqrt{K_p} = 1 + (0.1)\left(\frac{4}{5.7}\right)\sqrt{2.46} = 1.11$$

$$F_{qs} = F_{\gamma s} = 1 + (0.1)\left(\frac{B}{L}\right)K_p = 1 + (0.1)(\frac{5.7}{5.7})(2.46) = 1.25$$

The Meyerhof bearing capacity factors for $\phi = 25°$ *are*

$N_c = 20.7,\ N_q = 10.7,\ \text{and}\ N_\gamma = 6.77$

$q_{ult} = c' N_c (F_{cs} F_{cd} F_{ci}) + \bar{q} N_q (F_{qs} F_{qd} F_{qi}) + 0.5 \gamma B N_\gamma (F_{\gamma s} F_{\gamma d} F_{\gamma i})$

$q_{ult} = (0.24)(20.7)(1.49)(1.22)(1) + (0.112)(4)(10.7)(1.25)(1.11)(1) + (0.5)(0.112)(5.7)(6.67)(1.25)(1.11)(1)$

$q_{ult} = 18.6\ ksf$

$$q_{all} = \frac{q_{ult}}{FS} = \frac{18.6}{3} = 6.2\ ksf\ \text{therefore}\ B^2 = \frac{Q}{q_{all}} = \frac{200}{6.2} = 32.25\ ft^2 \therefore B = 5.7\ ft$$

Therefore the choice of $B = 5.7\ ft$ *was a good choice.*

(b) When the water table rises to the ground surface, need a larger footing; try $B = 7.0$ feet.

$$F_{cd} = 1 + 0.2\left(\frac{B}{L}\right)K_p = 1 + 0.2\left(\frac{7}{7}\right)(2.46) = 1.49$$

$F_{cs} = 1.49$ *same as above*

$$F_{qd} = F_{\gamma d} = 1 + 0.1\left(\frac{D}{B}\right)\sqrt{K_p} = 1 + 0.1\left(\frac{4}{7}\right)\sqrt{2.46} = 1.09$$

$F_{qs} = F_{\gamma s} = 1.25$ *same as above*

$q_{ult} = (0.24)(20.7)(1.49)(1.18) + (0.062)(4)(10.7)(1.25)(1.09) + (0.5)(0.062)(7)(6.67)(1.09)(1.25)$

$q_{ult} = 16.62\ ksf$

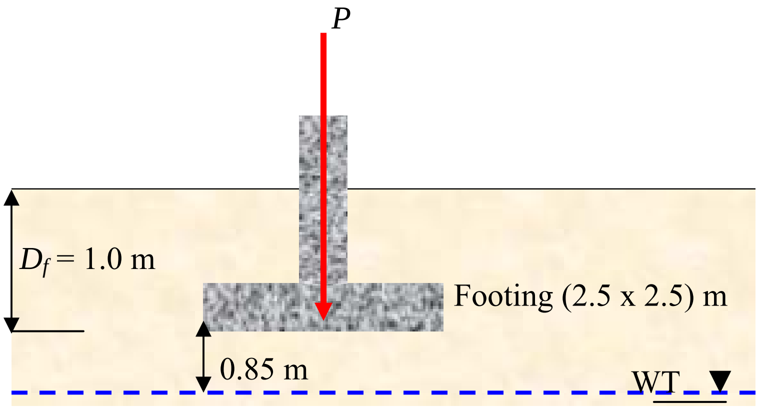
$$q_{all} = \frac{q_{ult}}{FS} = \frac{16.62}{3} = 5.54\ ksf\ \text{and}\ B^2 = \frac{Q}{q_{all}} = \frac{200}{5.54} = 36.1\ ft^2 \therefore B = 6.01\ ft$$

Iterate once more, and find $B = 7.5$ feet.

Bearing–08: *The effect of the WT upon the bearing capacity.*

(Revision: Sept-08)

Using the Hansen method, what are the ultimate and allowable bearing capacities for the footing shown below if you require a factor of safety of at least 2?



$c = 0$

$\phi_t = 35°$

$\gamma = 18.10 \frac{kN}{m^3}$

w_N = 10 %, and G_s = 2.68

Solution:

Always use the effective unit weight of water in the bearing capacity formulas. The average effective weight γ_e of the soil can also be given by the formula:

$$\gamma_e = \left(2H - d_w\right)\frac{d_w}{H^2}\gamma_{wet} + \frac{\gamma'}{H^2}\left(H - d_w\right)^2$$

$$where \quad H = (0.5)B\tan\left(45° + \frac{\varphi}{2}\right) = (0.5)(2.5)\tan\left(45° + \frac{35°}{2}\right) = 2.40\ m$$

$$and\ d_w = depth\ to\ the\ WT\ below\ the\ footing\ invert = 0.85\ m$$

$$Set\ the\ total\ volume\ V = 1\ m^3$$

$$\gamma_{dry} = \frac{\gamma_{wet}}{1+w} = \frac{18.10}{1+0.10} = 16.5\frac{kN}{m^3} \quad and \quad V_s = \frac{\gamma_{dry}}{G_s\gamma_{wet}} = \frac{16.5}{(2.68)(9.8)} = 0.63\ m^3$$

$$V_v = 1.0 - V_s = 1 - 0.63 = 0.37\ m^3 \quad and \quad \gamma_{sat} = \gamma_{dry} + n\gamma_{wet} = 16.5 + (0.37)(9.8) = 20.1\frac{kN}{m^3}$$

$$\therefore\ \gamma_e = ((2)(2.40 - 0.85))\left[\frac{0.85(18.10)}{(2.4)^2}\right] + \left[\frac{20.1 - 9.8}{(2.4)^2}\right](2.40 - 0.85)^2 = 12.6\frac{kN}{m^3}$$

Using Hansen's method with φ = 35°, the bearing capacity factors are N_q = 33.3 and $N_\gamma = 33.92$.

$$F_{qs} = 1 + \frac{B}{L}\tan\phi = 1 + \left(\frac{2.5}{2.5}\right)\tan 35° = 1.70$$

$$F_{qd} = 1 + 2\tan\phi\left(1 - \sin\phi\right)^2\left(\frac{D_f}{B}\right) = 1 + 2\tan 35^{\circ}\left(1 - \sin 35^{\circ}\right)^2\left(\frac{1}{2.5}\right) = 1.10$$

$$F_{\gamma s} = 1 - 0.4\frac{B}{L} = 1 - 0.4\left(\frac{2.5}{2.5}\right) = 0.6$$

$$F_{\gamma d} = 1.0$$

Therefore, the ultimate and allowable bearing capacities are,

$$q_{ult} = 0 + \overline{q}N_q(F_{qs}F_{qd}) + 0.5\gamma_e BN_\gamma(F_{\gamma s}F_{\gamma d})$$

$$q_{ult} = (18.1)(1.0)(33)(1.70)(1.10) + (0.5)(12.6)(2.5)(34)(0.6)(1)$$

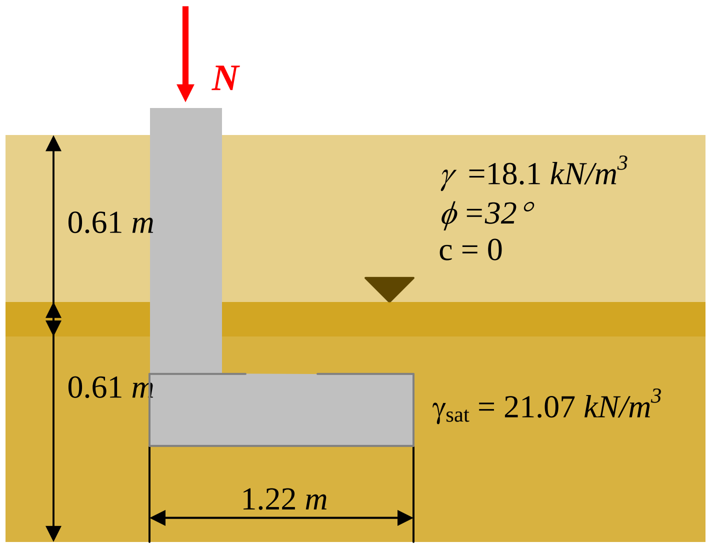
$$q_{ult} = 1{,}497 \ kPa$$

$$q_{all} = \frac{1{,}497}{2} = 749 \ kPa$$

*Bearing–09: *Finding the gross load capacity.*

(Revision: Sept-08)

Use the Hansen formula to determine the gross normal load N on the column shown below using a factor of safety of 3.



Solution:

The Hansen formula for a footing is,

$$q_{ult} = cN_c\, F_{cs}F_{cd} + qN_q\, F_{qs}F_{qd} + 0.5\gamma BN_\gamma\, F_{\gamma s}F_{\gamma d}$$

The inclination factors F_{ci}, F_{qi}, and $F_{\gamma i}$ are all equal to 1 because the load is vertical.

For ϕ = 32° , N_c = 35.49, N_q = 23.18 and N_γ = 20.79 and $B\ /\ L$ = 1

$$F_{cs} = 1 + \left(\frac{N_q}{N_c}\right) = 1 + (23.20/35.50) = 1.65$$

$$F_{qs} = 1 + \tan\phi = 1 + 0.62 = 1.62$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4 = 0.60$$

$$F_{qd} = 1 + 2\tan\phi(1-\sin\phi)^2\left(\frac{D_f}{B}\right) = 1 + (2)(0.62)(0.22)(1) = 1.273 \text{ for } D_f / B \le 1$$

$$F_{\gamma d} = 1$$

$$F_{cd} = F_{qd} - \left[\frac{(1-F_{qd})}{N_q\tan\phi}\right] = 1.273 - \left[\left(\frac{1-1.273}{23.20\times 0.62}\right)\right] = 1.292$$

The WT is located above the footing, therefore,

$$\bar{q} = (0.61m)(18.1\, kN/m^3) + (0.61m)(21.07 - 9.81) = 17.9\, kN/m^2$$

$$\therefore q_{ult} = (17.9)(1.62)(1.273)(23.20) + (0.5)(0.6)(21.07 - 9.81)(1.22)(20.8)(1) = 981\, kN/m^2$$

Therefore,

$$q_{all} = \left(\frac{q_{ult}}{3}\right) = \left(\frac{981 kN/m^2}{3}\right) = 327\, kN/m^2$$

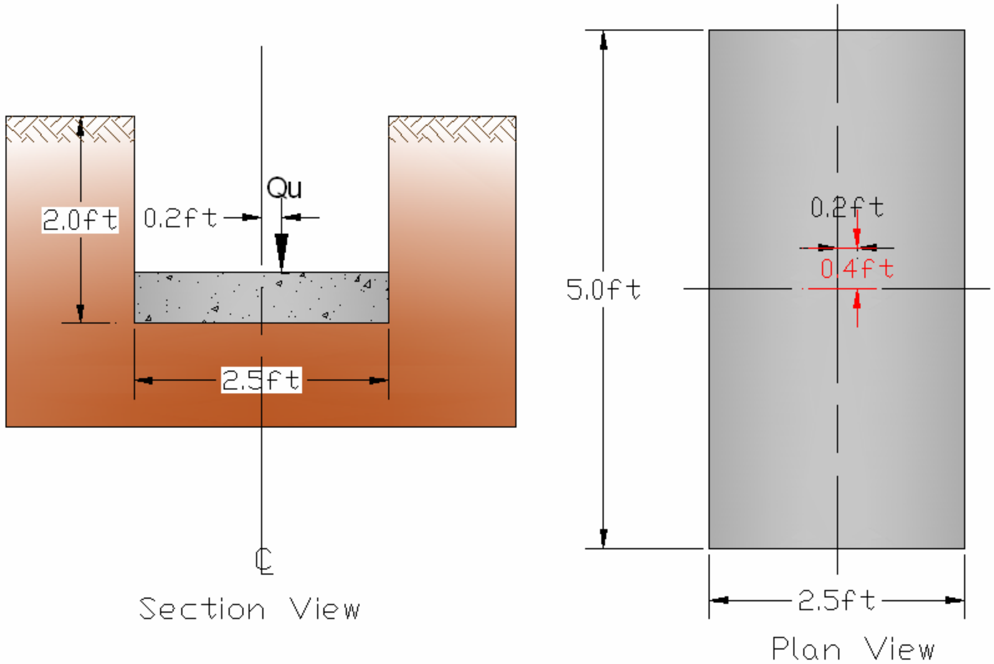
Hence, the total gross load N is,

$$N = q_{all} B^2 = (327 kN/m^2)(1.22m)^2 = 487\, kN$$

***Bearing–10:** The effect of an eccentric load upon bearing capacity.*

(Revision: Sept-08)

A rectangular footing measures 5 feet by 2.5 feet. Determine the gross ultimate load Q_{ult} applied eccentrically upon the footing, and the ultimate bearing capacity of the soil q_{ult}, given that γ = 115 pcf, c = 0 and ϕ = 30°.



Solution:

The effective footing width $B' = B - 2e_x = (2.5) - 2(0.2) = 2.1$ ft

and the effective length $L' = L - 2e_y = (5) - 2(0.4) = 4.2$ ft.

Meyerhof's ultimate bearing capacity formula with c = 0 is,

$$q_{ult} = 0 + \overline{q}N_q\ F_{qs}F_{qd} + 0.5\gamma B'N_\gamma\ F_{\gamma s}F_{\gamma d}$$

$$For\ \phi = 30^\circ, N_q = 18.4\ and\ N_\gamma = 15.67$$

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi = 1 + \left(\frac{2.1}{4.2}\right)(0.58) = 1.29$$

$$F_{qd} = 1 + 2(\tan 30^\circ)(1 - \sin 30^\circ)^2\left(\frac{2}{2.1}\right) = 1.275$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{2.1}{4.2}\right) = 0.8$$

$$F_{\gamma d} = 1$$

$$q_{ult} = (2)(0.115)(18.4)(1.29)(1.275) + (0.5)(0.115)(2.1)(15.67)(0.8)(1) = 8.47\ ksf$$

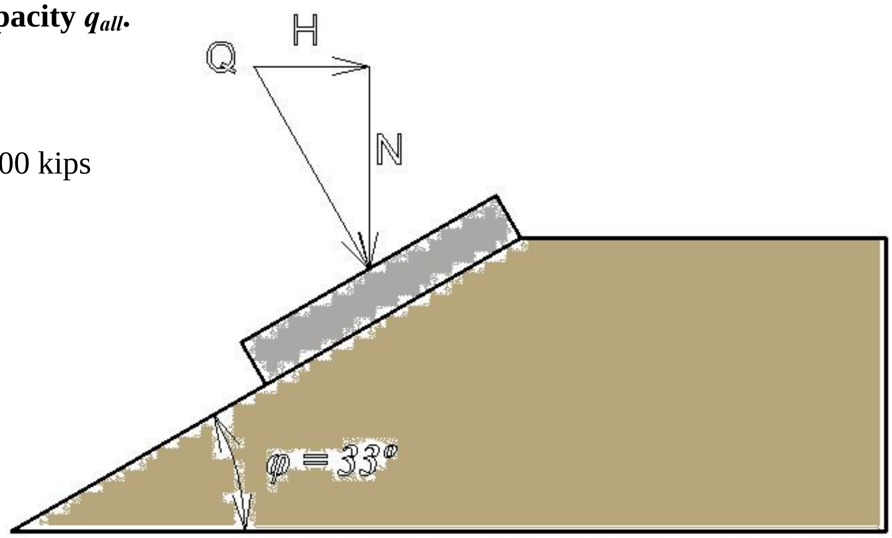
$$Hence, \quad Q_{ult} = q_{ult}(B'L') = (8.47)(2.1)(4.2) = 74.73\ kips$$

Bearing–11: *The effect of an inclined load upon the bearing capacity.*

(Revision: Sept-08)

A square 8' x 8' footing is loaded with an axial load of 400 kips and M_x = 200 ft-kips, M_y = 120 ft-kips. Un-drained triaxial tests (the soil is not saturated) gave φ = 33º and c = 200 psf. The footing depth D_f = 6.0 feet, the soil unit weight is 115 pcf, and the *WT* was not found. Use the *Hansen* equation with the *Meyerhof* reduction factors and a *FS = 3* to find the allowable bearing capacity q_{all}.

Vertical axial load = 400 kips
M_x = 200 ft-kips
M_y = 120 ft-kips



Solution:

Eccentricities $e_x = \dfrac{M_y}{Q} = \dfrac{120\,ft-k}{400} = 0.3\ feet$ and $e_y = \dfrac{M_x}{Q} = \dfrac{200\,ft-k}{400} = 0.5\ feet$

$\therefore\ B_r = B - 2e_y = 8' - 1' = 7\ feet$ and $L_r = L - 2e_x = 8' - 0.6' = 7.4\ feet$ (ie. $L_r > Br$)

Adjusting the ϕ from triaxial (ϕ_{tr}) to a plane-strain value (ϕ_{ps}) via Lee's formulation,

$$\phi_{ps} \cong 1.1\phi_{tr} = 1.1(32.7°) = 36°$$

$$N_q = e^{\pi \tan 36°}\tan^2\left(45° + \frac{36°}{2}\right) = 37.8$$

$$N_c = (N_q - 1)\cot\phi = (36.8)\cot 36° = 50.6$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi) = (36.8)\tan 50.4° = 44.4$$

$$N_\gamma = 1.5(N_q - 1)\tan\phi = 1.5(36.8)\tan 36° = 40.1$$

$$\therefore\quad S_c = 1 + 0.2K_p\left(\frac{B_r}{L_r}\right) = 1 + 0.2(3.85)\frac{7}{7.4} = 1.73$$

$$and\quad d_c = 1 + 0.2\sqrt{K_p}\left(\frac{D}{B_r}\right) = 1 + 0.2\left(\sqrt{3.85}\right)\left(\frac{6}{7}\right) = 1.34$$

Since $\phi > 10°$, $S_q = S_\gamma \cong 1.0$ *and* $d_q = d_\gamma = 1.0$.

Hansen's $q_{ult} = 0.5\gamma BN_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma + cN_c S_c d_c i_c g_c b_c + q_q N_q S_q d_q i_q g_q b_q$

Also $i = g = b = 1.0$ for this problem, since $\alpha = 0 =$ i (inclination factor f / load Q with t vertical) $\eta = 4$
g (ground factor with *t* inclined ground on side of footing)
b (base factor with t inclined ground under the footing)

$$q_{ult} = 0.5(0.115)(7)(40.1)(1) + (0.200)(50.6)(1.73)(1.34) + (0.115)(6)(37.8)(1) =$$
$$q_{ult} = 16.1 + 23.5 + 26.1 = 65.7 \ ksf$$

$$q_{all} = \frac{q_{ult}}{FS} = \frac{65.7}{3} = 21.9 \ ksf$$

$$R_{ex} = 1 - \left(\frac{e_x}{B}\right)^{\frac{1}{2}} = 1 - \left(\frac{0.3}{8}\right)^{\frac{1}{2}} = 0.81$$

$$R_{ey} = 1 - \left(\frac{e_y}{B}\right)^{\frac{1}{2}} = 1 - \left(\frac{0.5}{8}\right)^{\frac{1}{2}} = 0.75$$

$$Q_{all} = q_{all}\left(B^2\right)\left(R_{ex}\right)\left(R_{ey}\right) = 21.9(8x8)(0.81)(0.75) = 851 \ kips$$

$$q_{all} = \frac{Q_{all}}{B^2} = \left(\frac{851}{64}\right) = 13.3 \ ksf$$

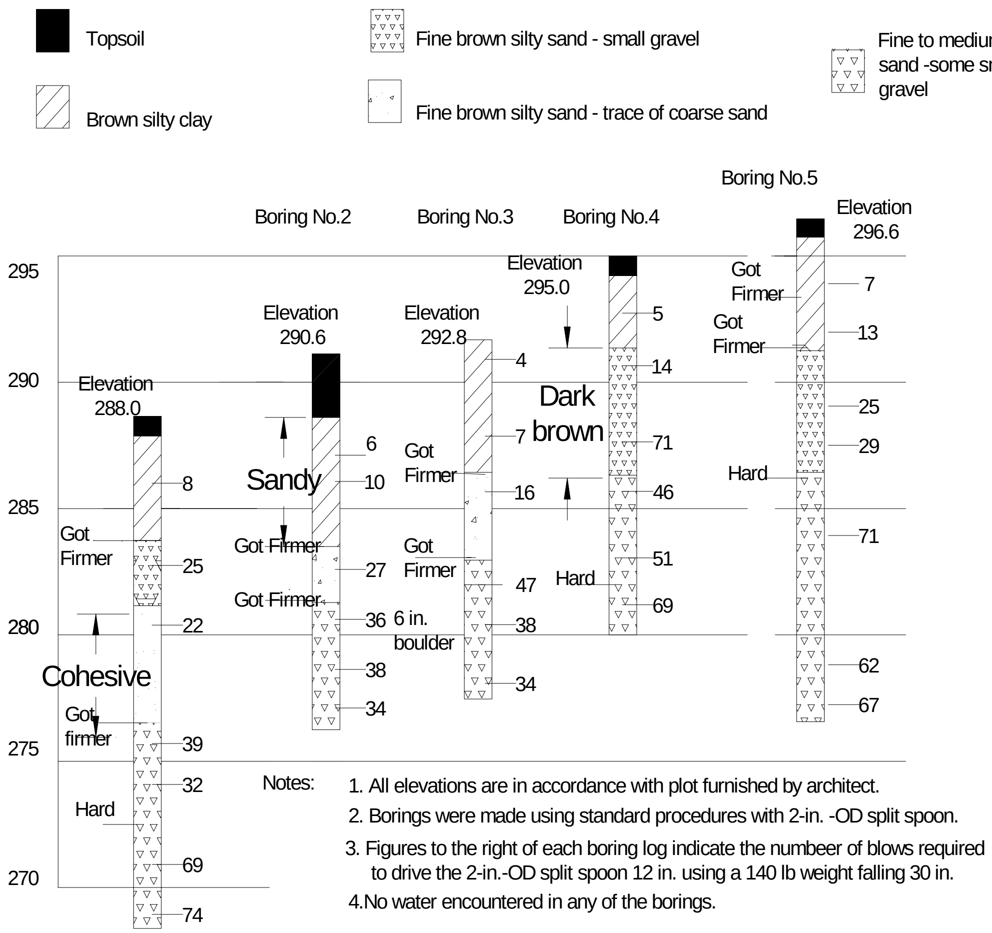
(The contact load $q_o = 13\left(\frac{400}{851}\right) = 6.1 \ ksf$)

Bearing-12: *Interpretation of borings to estimate a bearing capacity.*

(Revision: Sept-08)

Use the boring logs show below to recommend an allowable soil pressure q_{all} for the footings located in the vicinity of elevation 284, boring No. 2?

The building is a four-story (five on the low side) office building with column loads around 160 kips. State your reasons.



Solution:

It is presumed that all the building's footings will be placed at roughly elevation 284 or thereabouts. This is fine for the building area covered by borings # 3, 4 and 5 because they have good SPT values.

Meyerhof has proposed formulas for the allowable bearing capacity adjusted so that the settlement is limited to 1-inch. These formulas are:

$$q_{all} = \frac{N}{4}(K_D) \text{ for B} \leq 4 \text{ ft}$$

$$q_{all} = \frac{N}{6}\left(\frac{B+1}{B}\right)^2 (K_D) \text{ for B> 4ft}$$

where $K_D = 1+0.33\left(\frac{D_f}{B}\right) \leq 1.33$

For the silty sand use N= $\frac{47+51+71}{3} = 56.33 \approx 56$ (#3, 4, and 5)

Let's assume B=4.5 ft and D_f =0

$q_{all} = \frac{56}{6}\left(\frac{4.5+1}{45}\right)^2 (1) = 13.9\ ksf$ This suggests that a B = 4.5 feet is excessive since

$q_0 = \frac{Q}{B^2} = \frac{160kips}{20.25sf} = 7.9 \ll q_{all} = 13.9$ ksf

Assume B < 4 ft, say B~ 3.5 ft , and use formula

$q_{all} = \frac{N}{4}(K_D)$ $k_d = 1 + 0.33D_f/B$ $\frac{56}{4}\left(1+\frac{0.33D_f}{B}\right)$ and $D_f = 0$ $q_{all} = 14$ ksf

$\therefore\ q_o = \frac{Q}{B^2} = \frac{160\ kips}{(3.5)^2}$ = 13.06 ≅ 13 ksf ≤14 ksf OK

For footings in area of borings # 1 and #2, they will be deeper by 1-story (ie. for 5-story building). That places the shallow foundation at elevation 274 ft. This area will have bearing in the same strata. N= 32 and using B= 3.50' and D_f = 4.5'

$q_{all} = \frac{N}{4}\ k_d$ $k_d = 1+0.33D_f/B \leq 1.33$ $K_d = \frac{32}{4}\left(1+\frac{0.33x4.5}{3.50}\right) = 1.33$

$q_{all=} 10.64ksf < 13$ ksf NOT GOOD

Let's use B= 3.90 feet $q_{all} = 10.64$ ksf $q_0 = \left(\frac{Q}{B^2}\right) = 10.51$ ksf ≤ 10.64 ksf

Use ***B = 3.90 feet.***